Machine Design-I

(Session-2018-19)

Design Against Static Loading

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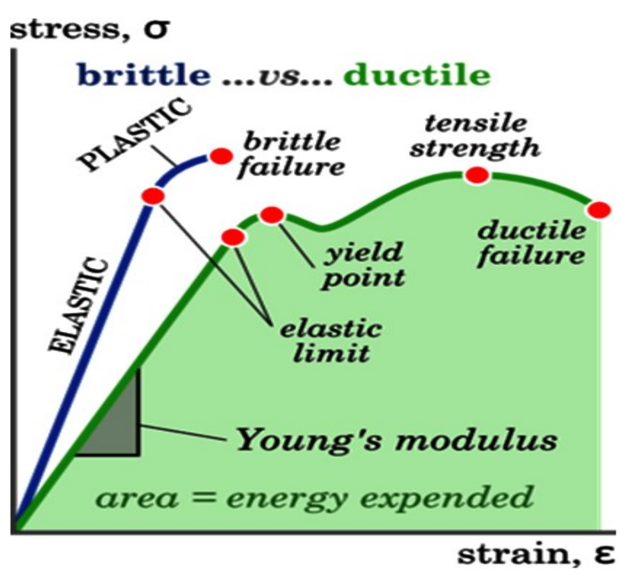
Design against static load

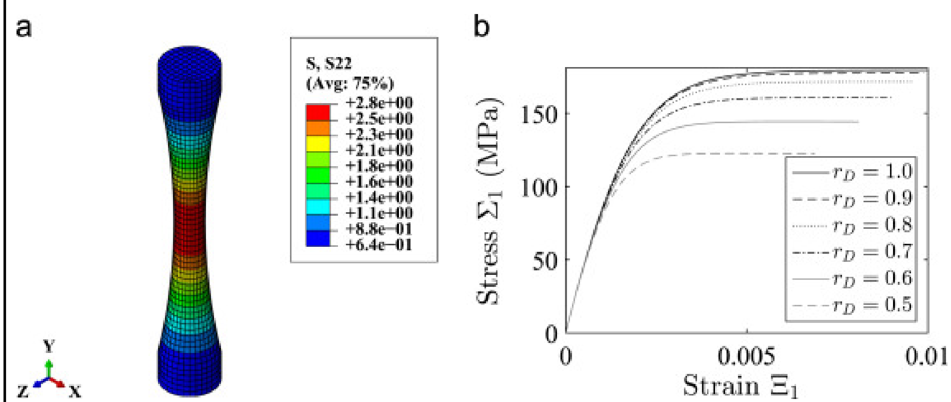
- A static load is defined as a force which is gradually applied to a mechanical component and which does not (or gradually) change its magnitude or direction with respect to time.
- Engineering Materials are classified in two groups –

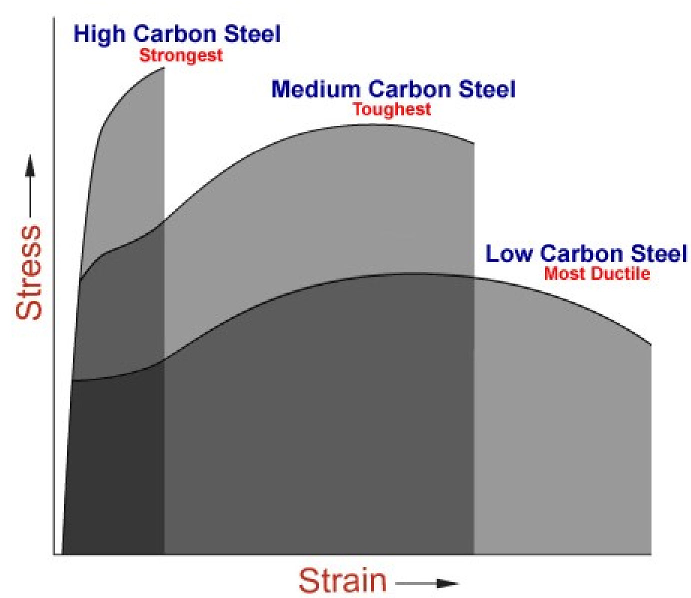
Ductile and brittle

- Ductile has large tensile strain before fracture.
  6% more than brittle materials.

Stress – strain curves for ductile and brittle materials




a
S, S22
(Avg: 75%)
+2.8e+00
+2.5e+00
+2.3e+00
+2.1e+00
+1.8e+00
+1.6e+00
+1.4e+00
+1.1e+00
+8.8e−01
+6.4e−01
Y
Z
X
b
Stress Σ_1 (MPa)
Strain Ξ_1
0
50
100
150
0
0.005
0.01
$r_D = 1.0$
$r_D = 0.9$
$r_D = 0.8$
$r_D = 0.7$
$r_D = 0.6$
$r_D = 0.5$


High Carbon Steel
Strongest
Medium Carbon Steel
Toughest
Low Carbon Steel
Most Ductile
Stress
Strain

Modes of failure

Failure by elastic deflection –

- Ex. Transmission shafts for gears.
- Lateral or torsional rigidity is design criterion.
- Elastic deflection also results in buckling of columns or vibrations.
- Design should be based on permissible **lateral and torsional deflections.**

Modes of failure (cont…..)

Failure by general yielding –

- Ductile material looses its usefulness due to large amount of plastic deformation after yield point stress reached.
- Considerable portion is subjected to plastic deformation is general yielding.
- Design should be based on permissible **Yield Strength.**

Modes of failure (cont…..)

Failure by fracture

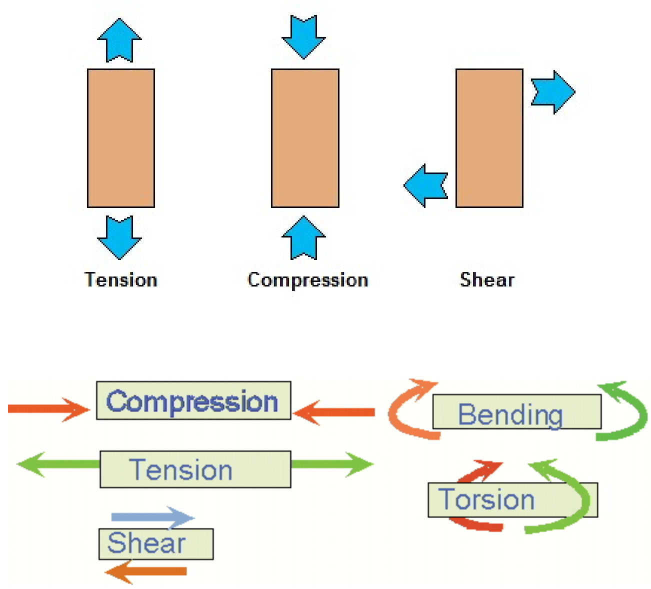
- Brittle materials fail because of the sudden fracture without any plastic deformation.
- The failure is sudden and total.
- Design should be based on permissible **Ultimate Strength.**

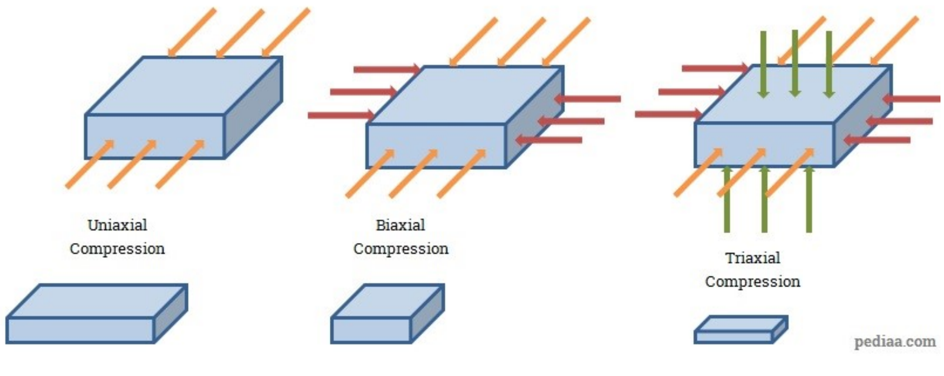
Conclusion

Strength ≥ Stress/ deformation/ Failure criterion

Stress – strain relationship

- When a mechanical component is subjected to an external static force, a resisting force is setup within the component.
- Internal resisting force per unit area is called as stress.
- The stresses are tensile if fibers of component tend to elongate.
- The stresses are compressive if fibers of component tend to shorten.





Stress – strain relationship

$$\sigma_t = \frac{P}{A}$$

Where σ_t = tensile stress
P = external force
A = cross-sectional area

$$\varepsilon = \frac{\delta}{l}$$

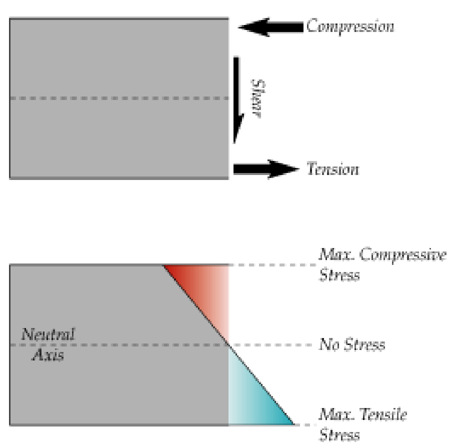
ϵ = strain
δ = elongation of tension rod
l = original length of rod

$$\delta = \frac{Pl}{AE}$$

The following assumptions should be made

1. The material is homogeneous.
2. The load is gradually applied.
3. The line of action of force P passes through the geometric axis of the cross – section.
4. The cross – section is uniform and free from the effects of stress concentration.

Stresses due to bending moment



Stresses due to bending moment

The bending stress in any fiber is given by

$$\sigma_b = \frac{M_b y \max}{I} \qquad \boldsymbol{\sigma \ \alpha \ y}$$

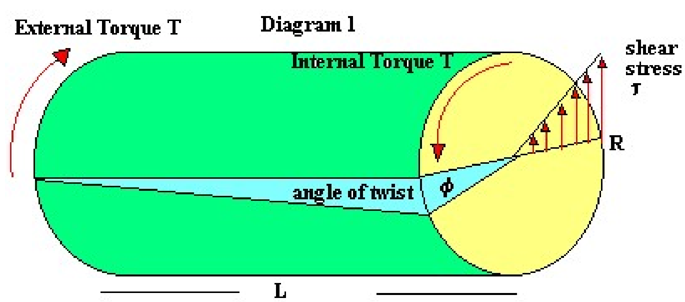
where

σ_b = bending stress at a distance of y from neutral axis (N/mm^2)

M_b = applied bending moment

I = moment of inertia of cross-section about the neutral axis (mm^4)

Stresses due to Torsional Moment (Torque)

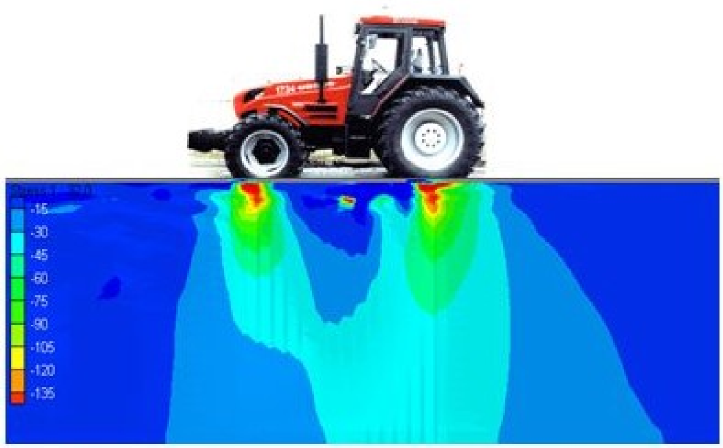


Stresses due to torsional moment

The internal stresses, which are induced resist the action of twist, are called torsional shear stresses.

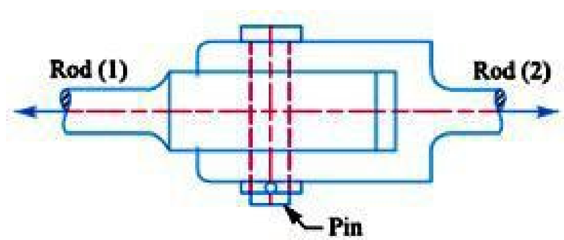
$$\tau = \frac{M_t r}{J}$$

FEM Modelling of stress propagation in ANSYS



Problem no.1

- **Two circular rods of 50 mm diameter are connected by a knuckle joint, as shown in Figure, by a pin of 40 mm in diameter. If a pull of 120 kN acts at each end, find the stress in the rod and pin.**



Factor of safety (Safe-Fail Design)

Number of factors which are difficult to evaluate accurately in design analysis.

- Uncertainty in magnitude of external force acting on component
- Variations in property of material like yield strength or ultimate strength
- Variations in dimension of component due to imperfect workmanship.

- While designing a component, it is necessary to ensure sufficient reserve strength.
- It is ensured by factor of safety (FS).

$$\text{FS} = \frac{\text{failure stress}}{\text{allowable stress}}$$

or

$$\text{FS} = \frac{\text{failure load}}{\text{working load}}$$

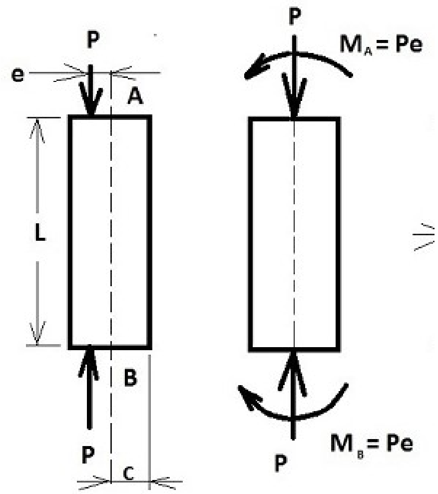
- For ductile materials $\sigma = \frac{\sigma_{yt}}{FS}$
- For brittle materials $\sigma = \frac{\sigma_{ut}}{FS}$

Selection of Factor of safety

Depends upon two factors

- Material of component
- Strength – criterion used

Eccentric Axial Loading



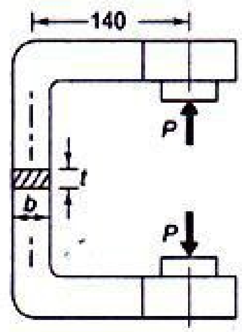
Eccentric axial loading (cont…)

- If the line of action of force does not pass through the centroid of cross – section. The loading is said to be an eccentric loading.
- The resultant stress at the cross – section

$$\sigma = \frac{P}{A} \pm \frac{Pey}{I}$$

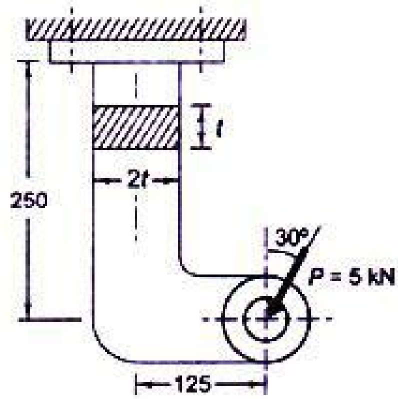
Problem no.2

- **Figure shows a C-clamp, which carries a load of 25 KN. The cross-section of the clamp is rectangular and the ratio of the width to thickness (b/t) is 2:1. The clamp is made of cast steel of grade 20-40 (Sut=400 N/mm²) and the factor of safety is 4. Determine the dimensions of the cross-section of the clamp.**



Problem no.3

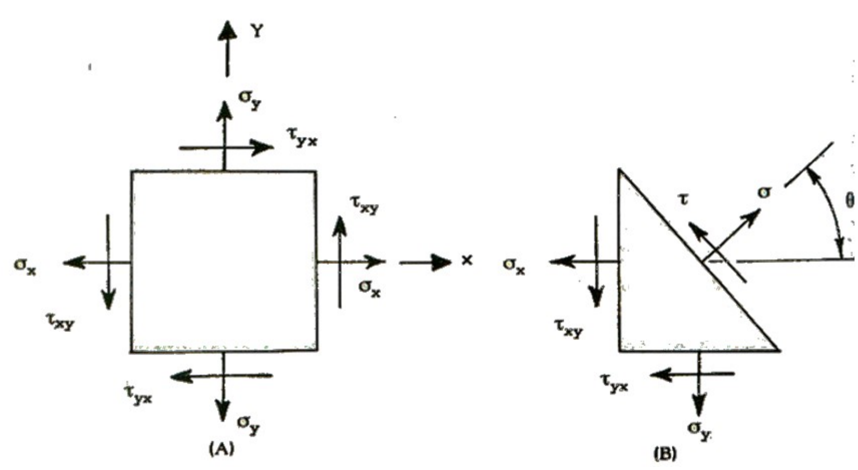
- **A bracket, made of steel FeE (Syt=200 N/mm²) and subjected to a force of 5 KN acting at an angle of 300 to the vertical, is shown in Figure. The factor of safety is 4. Determine the dimensions of the cross-section of the bracket.**



Principal stresses

- There are two types of stresses –
  Normal stresses (σ_x, σ_y, σ_z) and shear stresses (τ_{xy}, τ_{yx}).
- Predicting failure in members subjected to uniaxial stress is simple and straight-forward.
- But the problem of predicting the failure stresses for members subjected to **bi-axial, tri-axial stresses or combination of normal and shear stresses (e.g. a transmission shaft) is much more complicated. For design, it is necessary to determine the state of stresses (principal**

Two dimensional state of stress



$$\sigma = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

and

$$\tau = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Principal stresses

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\sigma_2 = \left(\frac{\sigma_x + \sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

Principal shear stress

$$\tau_{\max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$$\tan\theta_s = -\left(\frac{\sigma_x - \sigma_y}{2\tau_{xy}}\right)$$

Theories of failure (Application of principal stresses)

The principal theories of failure for a member subjected to bi-axial stress are as follows:

1. **Maximum principal (or normal) stress theory (Rankine's theory).**
2. **Maximum shear stress theory (Guest's or Tresca's theory).**
3. Maximum principal (or normal) strain theory (Saint Venant theory).
4. Maximum strain energy theory (Haigh's theory).
5. **Maximum distortion energy theory (Hencky and Von Mises theory)**

Maximum principal or normal stress theory (Rankine's theory)

- The failure or yielding occurs at a point in a member when the maximum principal or normal stress in a bi-axial stress system reaches the limiting strength of the material in a simple tension test.
- The limiting strength for ductile materials is yield point stress and for brittle materials the limiting strength is ultimate stress.

Maximum Principal or Normal Stress Theory (**Rankine's Theory**)

$\sigma_{t1} = \frac{\sigma_{yt}}{FS}$, for ductile materials

$\sigma_{t1} = \frac{\sigma_{u}}{FS}$, for brittle materials

σ_{yt} = Yield point stress in tension as determined from simple tension test.

σ_{u} = Ultimate stress.

- It ignores the possibility of failure due to shearing stress, therefore it is not used for ductile materials.
- However, for brittle materials which are relatively strong in shear but weak in tension or compression, this theory is generally used.

Maximum Shear Stress Theory
(Guest's or Tresca's Theory)

$$\tau_{max} = \frac{\tau_{yt}}{FS}$$

where τ_{max} = maximum shear stress in a bi-axial stress system

τ_{yt} = Shear stress at yield point as determined from simple tension test

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times FS}$$

This theory is mostly used for designing members of ductile materials.

Maximum Distortion Energy Theory
(Hencky and Von Mises Theory)

- The failure or yielding occurs at a point in a member when the distortion strain energy (also called shear strain energy) per unit volume in a bi-axial stress system reaches the limiting distortion energy (i.e. distortion energy at yield point) per unit volume as determined from a simple tension test.

- The maximum distortion energy theory for yielding is

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \sigma_{t1} \times \sigma_{t2} = \left(\frac{\sigma_{yt}}{FS}\right)^2$$

- This theory is mostly used for ductile materials.

In case of combined bending and torsional moments, there is a normal stress σ_x accompanied by the torsional shear stress τ_{xy}.

Substituting $\sigma_y = \sigma_z = \tau_{yz} = \tau_{zx} = 0$ in Eq.

$$\sigma = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

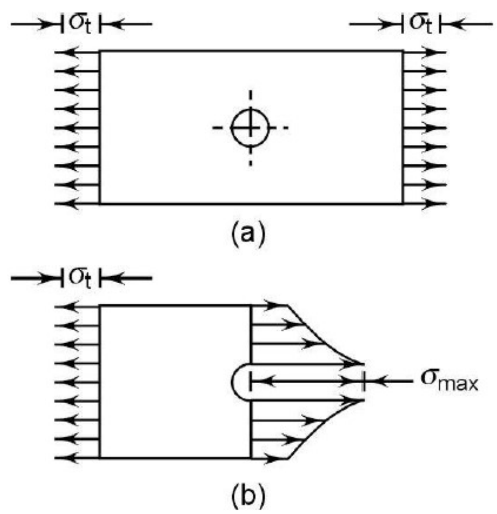
Problem no.4

The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to

1. Maximum principal stress theory;
2. Maximum shear stress theory;
3. Maximum distortion energy theory.

Take permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3.

Stress Concentration in static loading



Stress Concentration Factor (K_t)

Elementary equations:

$$\sigma_t = \frac{P}{A} \qquad \sigma_b = \frac{M_b\, y}{I} \qquad \tau = \frac{M_t\, r}{J}$$

Stress concentration is defined as the localization of high stresses due to **the irregularities** presents in the component and **abrupt changes** of the cross section.

Stress concentration factor (K_t) is defined as

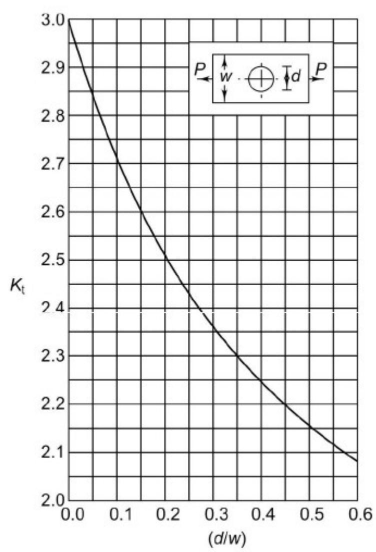
$$K_t = \frac{\text{highest value of actual stress near discontinuity}}{\text{nominal stresses obtained by elementary equations for minimal cross - section}}$$

Stress concentration factor for **Different Irregularities** (Follow any design **data book**)

The nominal stress is given by

$$\sigma_0 = \frac{P}{(w-d)t}$$

where t is the plate thickness.

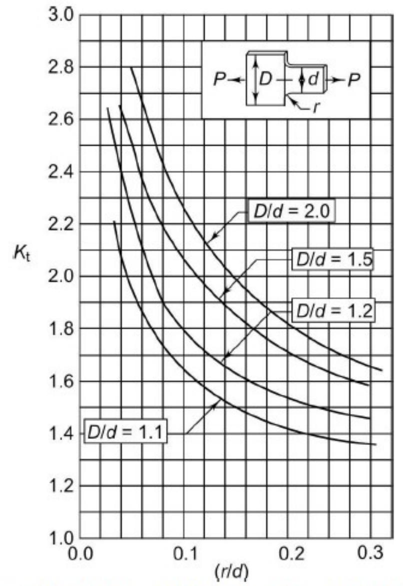


Stress Concentration Factor (Rectangular Plate with Transverse Hole in Tension or Compression)

The nominal stress is given by

$$\sigma_0 = \frac{P}{d\,t}$$

where t is the plate thickness.

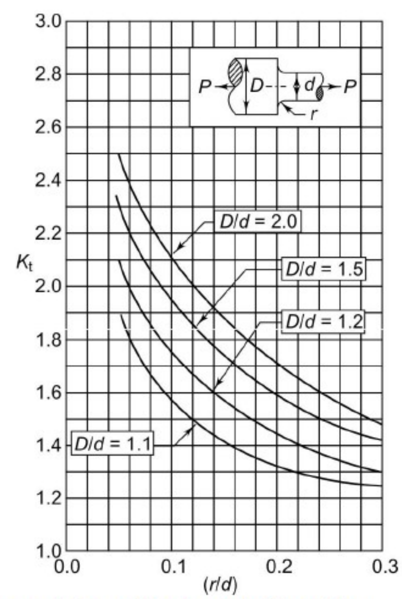


Stress Concentration Factor (Flat Plate with Shoulder Fillet in Tension or Compression)

The nominal stress is given by

$$\sigma_0 = \frac{P}{\frac{\pi}{4} d^2}$$

where d is the diameter on the small end.



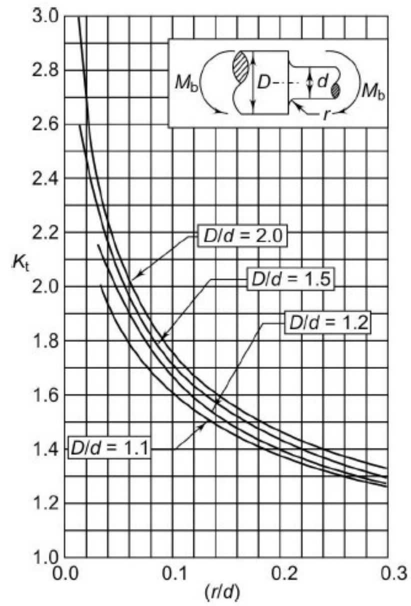
Stress Concentration Factor (Round haft with Shoulder Fillet in Tension)

The nominal stress is given by

$$\sigma_0 = \frac{M_b\, y}{I}$$

where d is the diameter on the smaller end.

$$I = \frac{\pi d^4}{64} \quad \text{and} \quad y = \frac{d}{2}$$



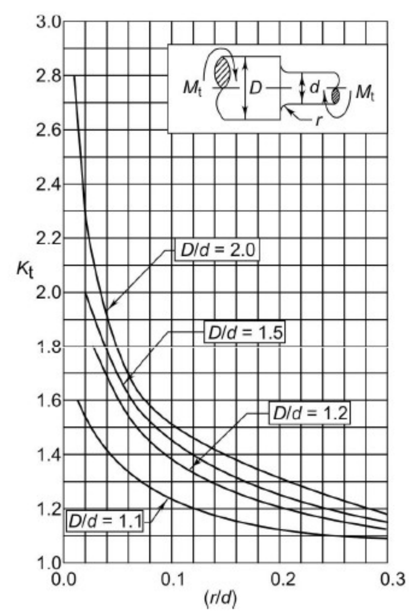
Stress Concentration Factor (Round Shaft with Shoulder Fillet in Bending)

The nominal stress is given by

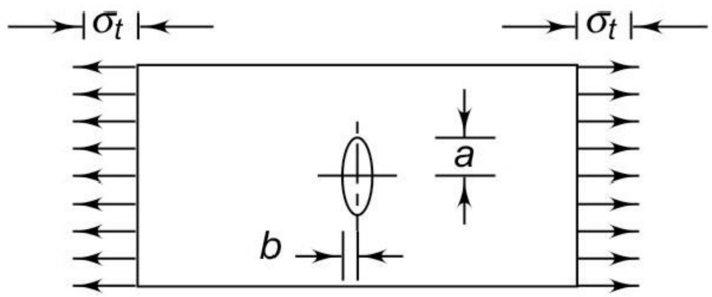
$$\tau_0 = \frac{M_t\, r}{J}$$

where d is the diameter on the smaller end.

$$J = \frac{\pi d^4}{32} \quad \text{and} \quad r = \frac{d}{2}$$



Stress Concentration Factor (Round Shaft with Fillet in Torsion)



$$K_t = 1 + 2(\frac{a}{b})$$

a = half width (or semi-axis) of ellipse perpendicular to the direction of load
b = half width (or semi-axis) of ellipse in the direction of load

Stress Concentration due to Elliptical Hole

Reduction of stress concentration

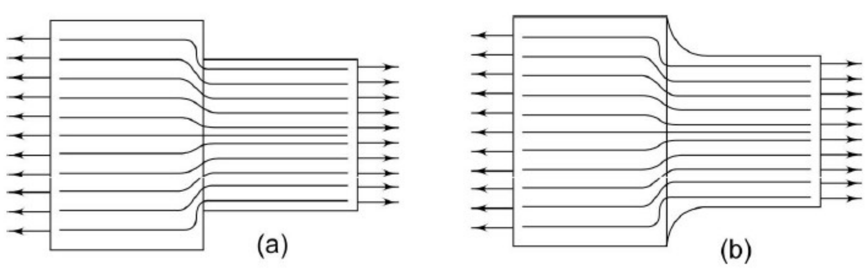


Fig. *Force Flow Analogy*
(a) Force Flow around Sharp Corner
(b) Force Flow around Rounded Corner

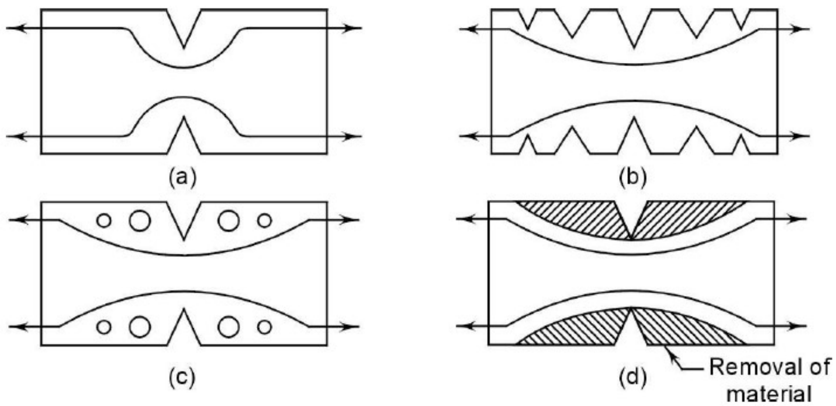


Fig. *Reduction of Stress Concentration due to V-notch*
(a) Original Notch (b) Multiple Notches
(c) Drilled Holes (d) Removal of Undesirable Material

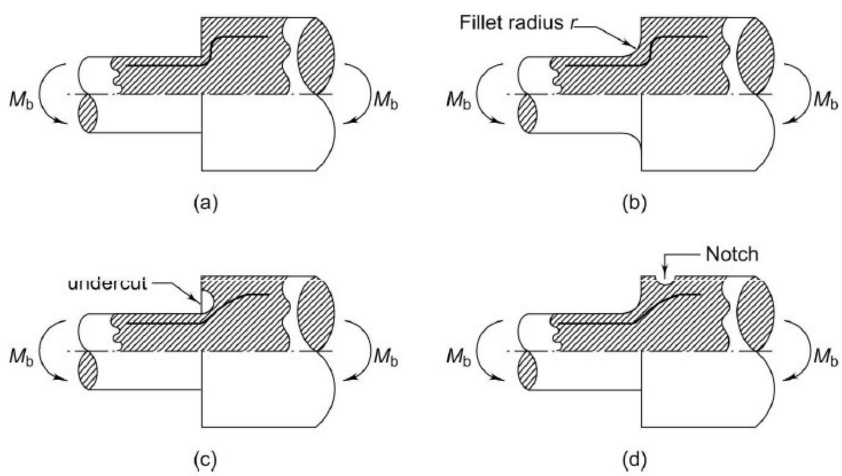


Fig. *Reduction of Stress Concentration due to Abrupt Change in Cross-section*
(a) Original Component (b) Fillet Radius
(C) Undercutting (d) Addition of Notch

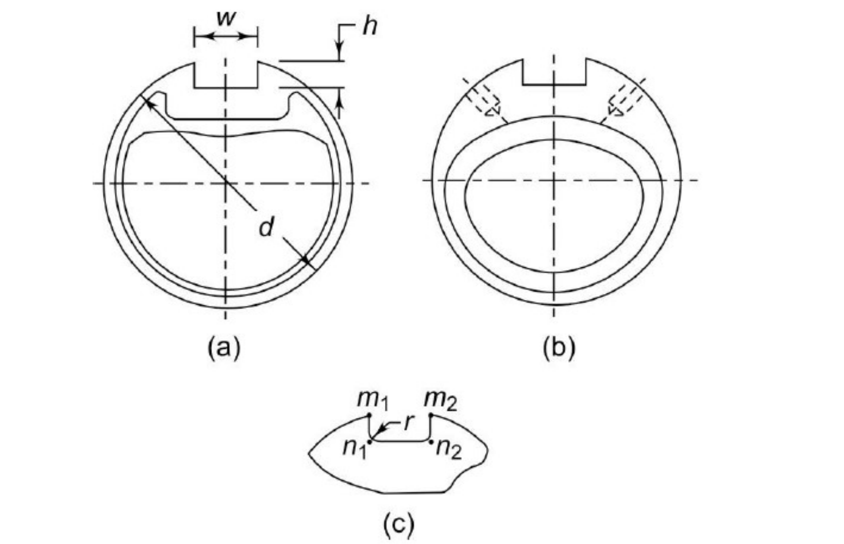


Fig. *Reduction of Stress Concentration in Shaft with Keyway (a) Original Shaft (b) Drilled Holes (c) Fillet Radius*

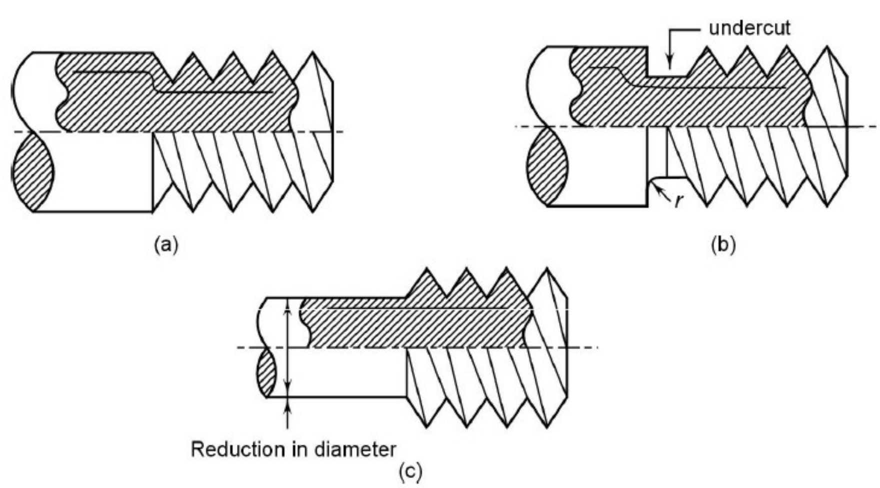


Fig. *Reduction of Stress Concentration in Threaded Components (a) Original Component (b) Undercutting (c) Reduction in Shank Diameter*